

**II B.Tech I Semester Supplementary Examinations, March 2006
MATHEMATICS-II**

(Common to Civil Engineering, Electrical & Electronic Engineering,
Mechanical Engineering, Electronics & Communication Engineering,
Computer Science & Engineering, Chemical Engineering, Electronics &
Instrumentation Engineering, Bio-Medical Engineering, Information
Technology, Electronics & Control Engineering, Mechatronics, Computer
Science & Systems Engineering, Electronics & Telematics, Metallurgy &
Material Technology, Electronics & Computer Engineering, Production
Engineering, Aeronautical Engineering, Instrumentation & Control
Engineering and Automobile Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Find the inverse of the matrix by elementary row transformations. [8]

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

- (b) Find whether the following equations are consistent, if so solve them.

$$x + 2y - z = 3$$

$$3x - y + 2z = 1$$

$$2x - 2y + 3z = 2$$

$$x - y + z = -1.$$

[8]

2. Diagonalize the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$ and hence find A^4 . [16]

3. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4yz - 8yz$ find the rank, index, signature and nature. [16]

4. (a) Expand $f(x) = e^{-x}$ as a Fourier series in $(-1, 1)$. [8]

- (b) Expand $x \sin x$ as sine series in $0 < x < \pi$. [8]

5. (a) Form the partial differential equation by eliminating the arbitrary function from $\phi\left(\frac{y}{x}, x^2 + y^2 + z^2\right) = 0$. [5]

- (b) Solve the partial differential equation $\frac{z^2}{p} + \frac{y^2}{q} = z$ [5]

- (c) Solve the partial differential equation $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$. [6]

6. A bar 10cm long with insulated surfaces, has its ends A and B kept at 40°C and 80°C , respectively until steady state conditions prevail. Then both the ends are suddenly insulated and kept so. Find the subsequent temperature function $u(x, t)$. [16]
7. Solve $\frac{\delta u}{\delta t} = \frac{\delta^2 u}{\delta t^2}$ for $x > 0, t > 0$, given that using Fourier transforms.
- (a) $u(0, t) = 0$ for $t > 0$
- (b) $u(x, 0) = \begin{cases} 1 & \text{for } 0 < x < 1 \\ 0 & \text{for } x \geq 1 \end{cases}$ and
- (c) $u(x, t)$ is bounded. [16]
8. (a) Prove that $Z(a^n f(t)) = F(z/a)$ [5]
- (b) Find
- i. $Z(-2)^n$ [5]
- ii. $Z(na^n)$
- (c) Find the inverse Z - transform of $\frac{z}{(z-1)(z-2)}$ [6]

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1. (a) Find the rank of the matrix by reducing it to the normal form. [8]

$$\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 1b & 4 & 12 & 15 \end{bmatrix}$$

- (b) Find whether the following set of equations are consistent if so, solve them. [8]

$$2x - y + 3z - 9 = 0$$

$$x + y + z = 6$$

$$x - y + z - 2 = 0$$

2. Show that the matrix $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ Satisfies its characteristic equation. Hence

Find A^{-1} [16]

3. (a) Show that $A = \begin{pmatrix} a + ic & -b + id \\ b + id & a - ic \end{pmatrix}$ is unitary matrix if $a^2 + b^2 + c^2 + d^2 = 1$. [8]

- (b) Find the eigen values of $\begin{pmatrix} 4 & 1 - 3i \\ 1 + 3i & 7 \end{pmatrix}$. [8]

4. Expand $f(x) = x \sin x, 0 < x < 2\pi$ as a Fourier series. [16]

5. (a) Form the partial differential equations by eliminating the arbitrary function $z = f(\sin x + \cos y)$. [5]

- (b) Solve the partial differential equation $zpq = p+q$. [6]

- (c) Solve the partial differential equation $(x-y)(px-xy) = (p-q)$. [5]

6. Solve the boundary value problem
 $u_{tt} = a^2 u_{xx}; 0 < x < L; t > 0$; with
 $u(0,t) = 0 = u(L,t)$; and $u(x,0) = x(L-x)$;
 $u_t(x,0) = 0$. [16]
7. Solve $\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}$, $0 < x < 4, t > 0$ using Fourier Transforms
given $u(0, t) = 0, u(4, t) = 0$,
 $u(x, 0) = 3\sin\pi x - 2\sin5\pi x$. [16]
8. (a) Find the Z - Transform of $C^k \cos\alpha k, k \geq 0$ [6]
(b) Using Z - transforms, solve the difference equation: $y_{n+2} + y_n = 2$
given that $y_0 = y_1 = 0$ [10]

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1. Find the values of λ for which the equations. [16]

$$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$$

$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$$

$$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$$

will have a nontrivial solution and solve them for each value of λ .

2. (a) If A and B are n rowed square matrices and if A is invertible show that $A^{-1}B$ and BA^{-1} have the same eigen values. [6]

- (b) Find the eigen values and the corresponding eigen vectors of the matrix. [10]

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

3. (a) By Lagrange's reduction transform the quadratic form $X^T A X$ to sum of

squares form for $A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 6 & -2 \\ 4 & -2 & 18 \end{pmatrix}$. [8]

- (b) Reduce the matrix $9x^2 + 2y^2 + 2z^2 + 6xy + 2yz - 2zx$ to a canonical form and find the rank, index and the signature. [8]

4. (a) Express $f(x) = |x|$, $-\pi < x < \pi$ as Fourier series. Hence show that [8]
- $$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$$

- (b) $f(x) = \begin{cases} \frac{\pi}{3}, & \text{for } 0 \leq x < \frac{\pi}{3} \\ 0, & \text{for } \frac{\pi}{3} \leq x < \frac{2\pi}{3} \\ \frac{-\pi}{3}, & \text{for } \frac{2\pi}{3} \leq x < \pi \end{cases}$ show that
- $$f(x) = \frac{2}{\sqrt{3}} \left[\cos x - \frac{\cos 5x}{5} + \frac{\cos 7x}{7} - \dots \right]$$
- [8]

5. (a) Form the partial differential equations by eliminating the arbitrary functions
 $Z = y^2 + 2f(1/x + \log y)$ [5]
- (b) Solve the partial differential equation $(x^2 - y^2 - z^2)p + 2xyq = 2xz$. [6]
- (c) Solve the partial differential equation $z^2(p^2x^2 + q^2) = 1$. [5]
6. Solve $\delta^2u/\delta x^2 + \delta^2u/\delta y^2 = 0$. Subject to the boundary conditions $u(0,y) = u(L,y) = u(x,0) = 0$ and $u(x,L) = \sin \pi x/L$. [16]
7. (a) State and prove Fourier Integral Theorem. [8]
- (b) Find the Fourier transform of $f(x) = \begin{cases} e^{ikx} & a < x < b \\ 0 & x < a \text{ and } x > b \end{cases}$ [8]
8. (a) Find $Z[a^n]$ [8]
- (b) Solve the difference equation, using Z - transforms $4u_n - u_{n+2} = 0$ given that $u_0 = 0, u_1 = 2$. [8]

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1. (a) For what value of K the matrix [8]

$$\begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix} \text{ has rank 3.}$$

- (b) Find whether the following set of equations are consistent if so, solve them. [8]

$$x_1 + x_2 + x_3 + x_4 = 0$$

$$x_1 + x_2 + x_3 - x_4 = 4$$

$$x_1 + x_2 - x_3 + x_4 = -4$$

$$x_1 - x_2 + x_3 + x_4 = 2.$$

2. (a) If λ is an eigen value of a non singular matrix A, show that

i. $\frac{1}{\lambda}$ is an eigen value of A^{-1} and hence deduce that [4]

ii. $\frac{|A|}{\lambda}$ is an eigen value of the matrix adj A. [2]

- (b) Find the eigen values and the corresponding eigen vectors of the matrix

$$\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

3. (a) By Lagrange's reduction transform the quadratic form $X^T A X$ to sum of

squares form for $A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 6 & -2 \\ 4 & -2 & 18 \end{pmatrix}$. [8]

- (b) Reduce the matrix $9x^2 + 2y^2 + 2z^2 + 6xy + 2yz - 2zx$ to a canonical form and find the rank, index and the signature. [8]

4. (a) Expand $f(x) = \cos ax$ as a Fourier series in $(-\pi, \pi)$ where a is not an integer.
Hence prove that $\cot\theta = \frac{1}{\theta} + \frac{2\theta}{\theta^2 - \pi^2} + \frac{2\theta}{\theta^2 - 4\pi^2} + \dots$ [8]

(b) If $f(x) = x$, $0 < x < \frac{\pi}{2}$

$$= \pi - x, \quad \frac{\pi}{2} < x < \pi$$

Show that $f(x) = \frac{4}{\pi} \left[\sin x - \frac{1}{3^2} \sin 3x + \frac{1}{5^2} \sin 5x - \dots \right]$ [8]

5. (a) Form the partial differential equation by eliminating the arbitrary function from $z = f\left(\frac{xy}{z}\right)$. [5]

(b) Solve the partial differential equation $p^2x + q^2y = z$ [5]

(c) Solve the partial differential equation $(1 + y)p + (1 + x)q = z$. [6]

6. A homogeneous rod of conducting material of length 100cm has its ends kept at zero temperature and the temperature initially is

$$u(x, 0) = x; \quad 0 \leq x \leq 50$$

$$= 100 - x; \quad 50 \leq x \leq 100.$$

Find the temperature $u(x, t)$ at any t . [16]

7. (a) Find the finite Fourier cosine transform of $f(x) = x^2$ in $(0, 1)$. [6]

(b) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$ ($a > 0$) and deduce that

$$\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x} \sin sx \, dx = \tan^{-1}\left(\frac{b}{s}\right) - \tan^{-1}\left(\frac{a}{s}\right)$$
 [10]

8. (a) Find $Z \left[n^2 3^n + 2 \cdot \frac{4^n}{n} \right]$ [8]

(b) Solve the difference equation, using Z -transforms $u_{n+2} - u_n = 2^n$ where $u_0 = 0$, $u_1 = 1$. [8]
