

II B.Tech I Semester Supplementary Examinations, February 2007
MATHEMATICS-II

(Common to Civil Engineering, Mechanical Engineering, Chemical Engineering, Mechatronics, Metallurgy & Material Technology, Production Engineering, Aeronautical Engineering and Automobile Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) For what value of K the matrix $\begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ K & 2 & 2 & 2 \\ 9 & 9 & K & 3 \end{bmatrix}$ has rank 3.
- (b) Find whether the following set of equations are consistent if so, solve them.
- $$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ x_1 + x_2 + x_3 - x_4 &= 4 \\ x_1 + x_2 - x_3 + x_4 &= -4 \\ x_1 - x_2 + x_3 + x_4 &= 2. \end{aligned} \quad [8+8]$$
2. (a) Find the eigen values and the corresponding eigen vectors of the matrix
- $$A = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$$
- (b) Prove that the two eigen vectors corresponding to the two different eigen values are linearly independent. [10+6]
3. (a) Use lagrange reduction method to reduce the quadratic form $x_1^2 + 6x_2^2 + 18x_3^2 + 4x_1x_2 + 8x_1x_3 - 4x_2x_3$ to the sum of squares and find the corresponding linear transformation also find the index and signature.
- (b) Show that $A = \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$ is orthogonal. [8+8]
4. (a) Given that $f(x) = x + x^2$ for $-\pi < x < \pi$ find the Fourier expansion of f(x). Deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$
- (b) Find the half range sine series for $f(x) = x(\pi - x)$, in $0 < x < \pi$. Deduce that $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots = \frac{\pi^3}{32}$. [10+6]
5. (a) Form the partial differential equation by eliminating the arbitrary function from $z = f\left(\frac{xy}{z}\right)$.
- (b) Solve the partial differential equation $(1 + y) p + (1 + x) q = z$.
- (c) Solve the partial differential equation $(y^2 + z^2) p - xyq = -xz$. [5+5+6]

6. Solve the boundary value problem $u_t = u_{xx}; 0 < x < \ell, t > 0$ with $u(0, t) = 0$;
 $u_x(\ell, t) = 0$ and $u(x, 0) = x$. [16]
7. (a) Find the finite Fourier sine and cosine transforms of
i. $f(x) = x$ in $(0, 1)$.
(b) Find the finite sine and transform of $f(x) = \cos kx$ in $0 < x < \pi$ [8+8]
8. (a) If $Z(n^2) = \frac{z^2+z}{(z-1)^3}$, find $Z(n^3)$ and $Z(n^4)$
(b) Using convolution theorem find $Z^{-1} \left[\frac{z^2}{(z-4)(z-5)} \right]$. [8+8]

II B.Tech I Semester Supplementary Examinations, February 2007**MATHEMATICS-II**

(Common to Civil Engineering, Mechanical Engineering, Chemical Engineering, Mechatronics, Metallurgy & Material Technology, Production Engineering, Aeronautical Engineering and Automobile Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Find the rank of the matrix by reducing it to the echlon form

$$\begin{bmatrix} 1 & 0 & -5 & 6 \\ 3 & -2 & 1 & 2 \\ 5 & -2 & -9 & 14 \\ 4 & -2 & -4 & 8 \end{bmatrix}$$

- (b) Show that the equations

$$3x + 4y + 5z = a, \quad 4x + 5y + 6z = b$$

$$5x + 6y + 7z = c, \text{ do not have a solution unless } a + c = 2b.$$

[8+8]

2. (a) Find the eigen values and the corresponding eigen vectors of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

- (b) Prove that the product of the eigen values is equal to the determinant of the matrix. [10+6]

3. (a) Use lagrange reduction method to reduce the quadratic form $x_1^2 + 6x_2^2 + 18x_3^2 + 4x_1x_2 + 8x_1x_3 - 4x_2x_3$ to the sum of squares and find the corresponding linear transformation also find the index and signature.

- (b) Show that $A = \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$ is orthogonal. [8+8]

4. (a) Expand $f(x) = e^{-x}$ as a Fourier series in $(-1, 1)$.

- (b) Expand $x \sin x$ as a sine series in $0 < x < \pi$. [10+6]

5. (a) Form the partial differential equation by eliminating the arbitrary function $z = f_1(y + 2x) + f_2(y - 3x)$.

- (b) Solve the partial differential equation $zpq = p + q$.

6. A taut string of length L is fastened at both ends. The midpoint of the string is taken to a height "b" and then released from rest in that position. Find the displacement of the string at any position x at any time t . [16]

7. (a) Using Fourier integral show that $\int_0^\alpha \frac{w \sin xw}{1+w^2} dw = \frac{\pi}{2} e^{-x} (x > 0)$

(b) Using Fourier integral show that $\int_0^\infty \frac{1-\cos \pi \lambda}{\lambda} \sin x \lambda d\lambda$
 $= \frac{\pi}{2}$ if $0 < x < \pi$
 $= 0$ if $x > \pi$ [8+8]

8. (a) Z - transform of $n \cos n \theta$.

(b) Find $Z^{-1} \left[\frac{1}{(z-5)^3} \right]$ $|z| > 5$. Determine the region of convergence. [6+10]

II B.Tech I Semester Supplementary Examinations, February 2007
MATHEMATICS-II

(Common to Civil Engineering, Mechanical Engineering, Chemical Engineering, Mechatronics, Metallurgy & Material Technology, Production Engineering, Aeronautical Engineering and Automobile Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$
- (b) Find whether the following set of equations are consistent if so, solve them.
 $2x - y + 3z - 9 = 0$
 $x + y + z = 6$
 $x - y + z - 2 = 0.$ [8+8]
2. (a) Find the eigen values and the corresponding eigen vectors of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
- (b) Prove that the product of the eigen values is equal to the determinant of the matrix. [10+6]
3. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ find the rank, index, signature and nature. [16]
4. (a) Express $f(x) = |x|$, $-\pi < x < \pi$ as Fourier series. Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$
- (b) $f(x) = \begin{cases} \frac{\pi}{3}, & \text{for } 0 \leq x < \frac{\pi}{3} \\ 0, & \text{for } \frac{\pi}{3} \leq x < \frac{2\pi}{3} \\ -\frac{\pi}{3}, & \text{for } \frac{2\pi}{3} \leq x < \pi \end{cases}$ show that $f(x) = \frac{2}{\sqrt{3}} \left[\cos x - \frac{\cos 5x}{5} + \frac{\cos 7x}{7} - \dots \right]$. [8+8]
5. (a) Form the partial differential equation by eliminating the arbitrary function from $f(x^2 + y^2 + z^2, xy + z)$.
- (b) Solve the partial differential equation $\frac{p}{x^2} + \frac{q}{y^2} = z$.
- (c) Solve the partial differential equation $x^2p + y^2q = (x + y)$. [5+5+6]
6. A square plate has its faces $x = 0$ and $x = \pi$ ($0 < y < \pi$) insulated. Its edges $y = \pi$ and $y = 0$ are kept at temperatures 0 and $f(x)$ respectively. Derive the formula for steady state temperature. [16]
7. (a) Using Fourier integral show that $\int_0^\alpha \frac{w \sin xw}{1+w^2} dw = \frac{\pi}{2} e^{-x} (x > 0)$

(b) Using Fourier integral show that $\int_0^\infty \frac{1-\cos \pi \lambda}{\lambda} \sin x \lambda d\lambda$
 $= \frac{\pi}{2}$ if $0 < x < \pi$
 $= 0$ if $x > \pi$ [8+8]

8. (a) Find the Z transform of $\sin (3n+5)$

(b) Find $Z^{-1} \left[\frac{z}{z^2+11z+24} \right]$. [8+8]

II B.Tech I Semester Supplementary Examinations, February 2007**MATHEMATICS-II**

(Common to Civil Engineering, Mechanical Engineering, Chemical Engineering, Mechatronics, Metallurgy & Material Technology, Production Engineering, Aeronautical Engineering and Automobile Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Determine the rank of the matrix.

$$A = \begin{pmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix} \text{ by reducing it to normal form.}$$

- (b) Find whether the following equations are consistent, if so solve them.

$$x + 2y - z = 3$$

$$3x - y + 2z = 1$$

$$2x - 2y + 3z = 2$$

$$x - y + z = -1.$$

[8+8]

2. (a) Find the eigen values and the corresponding eigen vectors of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

- (b) Prove that the product of the eigen values is equal to the determinant of the matrix. [10+6]

3. (a) Prove that every hermitian matrix can be written as $A + iB$ where A is real and Symmetric and B is real and Skew-Symmetric.

- (b) Reduce the quadratic form $x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ to a canonical form. [6+10]

4. (a) An alternating current after passing through rectifier has the form

$$i = \begin{cases} I_0 \sin x, & \text{for } 0 \leq x \leq \pi \\ 0, & \text{for } \pi \leq x \leq 2\pi \end{cases} \text{ where } I_0 \text{ is the maximum current and the period is}$$

2π . Express i as a Fourier series.

- (b) Represent the following function by Fourier sine series

$$f(x) = \begin{cases} 1, & 0 < x < \frac{m}{2} \\ 0, & \frac{m}{2} < x < m \end{cases} \quad [10+6]$$

5. (a) Form the partial differential equation by eliminating the arbitrary function from $f(x^2 + y^2 + z^2, xy + z)$.

- (b) Solve the partial differential equation $\frac{p}{x^2} + \frac{q}{y^2} = z$.

- (c) Solve the partial differential equation $x^2p + y^2q = (x + y)$. [5+5+6]

6. (a) Solve by separation of variables $3u_x + 2u_y = 0$ with $u(x,0) = 4e^{-x}$.

- (b) Obtain the general solution of the one dimension wave equation
 $\partial^2 u / \partial t^2 = c^2 \partial^2 u / \partial x^2$. [8+8]
7. (a) Find the finite cosine transform of $\{f(x) = \frac{\pi}{3} - x + \frac{x^2}{2\pi}$ in $(0, \pi)$
(b) Find the Fourier cosine transforms of $e^{-ax} \sin ax$. [8+8]
8. (a) State and Prove damping rule.
(b) Find Z (cos h at. sin bt)
(c) Find the inverse Z transform of $\frac{z}{z^2+7z+10}$. [5+6+5]
