Code No: Z0125 / R07 **Set No. 1**

I B.Tech - Regular Examinations, June 2009
MATHEMATICS-I
(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering and Bio-Technology)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Solve (xy sin xy + cos xy) y dx + (xy sinxy - cos xy) x dy = 0.
   (b) The world population at the beginning of 1970 was 3.6 billion. The weight of the earth is 6.586 $\times$ 10^{21} tons. If the population continues to increase exponentially, with a growth constant k = 0.02 and with time measured in years, in what year will the weight of all people equal the weight of the earth, if we assume that the average person weighs 120 pounds? [8+8]

2. Solve $\frac{d^4y}{dx^4}$ - y = cosx coshx + 2x^4 + x - 1. [16]

3. (a) Verify Rolle's theorem for f(x) = x^2 - 2x -3 in the interval (1,-3).
   (b) Prove that u = $\frac{x^2-y^2}{x^2+y^2}$, v = $\frac{2xy}{x^2+y^2}$ are functionally dependent and find the relation between them. [8+8]

4. (a) Find the radius of curvature of $x = ae^{\theta}[\sin\theta - \cos\theta]$, $y = ae^{\theta}[\cos\theta - \sin\theta]$ $at\ \theta = 0$.
   (b) Trace the curve y^2 (a-x) = x^3, (a>0) [8+8]

5. (a) Find the volume of the solid that results when the region enclosed by the curves xy = 1, x-axis and x = 1 rotated about x-axis.
   (b) Evaluate $\int\limits_0^1 \int\limits_x^{\sqrt{x}} x^2y^2(x+y)\, dydx$. [8+8]

6. (a) Test the convergence of
   $\sum (1 + \frac{1}{\sqrt{n}})^{-n^2}$
   (b) Test the convergence of
   $\sum$ (x^n /n) , (x > 0) [10+6]

7. Verify Stoke's theorem for $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\,\vec{j}$ over the box bounded by the planes $x = 0, x = a, y = 0, y = b, z = c$. [16]

8. (a) Find L [f (t)] where f (t) is a periodic function of 2 π and it is given by
   f (t) = sint, $0 < t < \pi$
   f (t) = 0, $\pi < t < 2\pi$.

   (b) Find L^{-1} [s/ (s^2 +4s +5)]. [8+8]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Solve {y(1+$\frac{1}{x}$)cosy} dx+ {x + logx − xsiny} dy=0. [8+8]
   (b) An object whose temperature is 75^0 C cools in an atmosphere of constant temperature 25^0 C at the rate of kθ where θ being the excess temperature of the body over that of the atmosphere. If after 10 minutes the temperature of the object falls to 65^0 C, find its temperature after 20 minutes and also find the time required to cool down to 55^0 C. [8+8]

2. (a) Solve (D^2 - 4D + 13) y = e^{2x}
   (b) Solve (D^2 + 16) y = e^{-4x}. [8+8]

3. (a) Verify Rolle's theorem for f(x) = x^2 - 2x -3 in the interval (1,-3).
   (b) Prove that u = $\frac{x^2-y^2}{x^2+y^2}$, v = $\frac{2xy}{x^2+y^2}$ are functionally dependent and find the relation between them. [8+8]

4. (a) Show that the evolute of the cycloid x = a(θ - sin θ), y = a(1 - cos θ) is another cycloid.
   (b) Trace the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. [8+8]

5. Evaluate $\iint\limits_R ye^{-(x+y)} \sin \frac{\pi y^2}{(x+y)^2} \quad dxdy$, where R is the infinite positive quadrant, by using the change of variables given as $u = x + y$, $\nu = y$. [16]

6. (a) Examine the convergence of
   $1 + 2(1/3) + (1/3)^2 + 2(1/3)^3 + (1/3)^4 + ...$
   (b) Discuss the convergence of
   $\frac{3x}{7} + \frac{3.6x^2}{7.10} + \frac{3.6.9x^3}{7.10.13} + ..., (x > 0)$ [6+10]

7. (a) Prove that $\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla \cdot \vec{F}) - \nabla^2\vec{F}$
   (b) Using Green's theorem in a plane to evaluate $\int\limits_C [(2x-y)dx + (x+y)dy]$, where C is the boundary of the circle $x^2 + y^2 = a^2$ in the xy-plane. [8+8]

8. (a) Using Laplace transform, solve $(D^2 + 2D - 3)y = \sin x$, $y(0) = y'(0) = 0$.

   (b) Using Laplace transform evaluate $\int_0^{\infty} (e^{-t} - e^{-2t})/t \, dt$. [8+8]

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Code No: Z0125 / R07 **Set No. 3**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Solve $\frac{dy}{dx} = \frac{y^3+3x^2y}{x^3+3xy^2}$.
   (b) Find the orthogonal trajectories of the family of curves $x^{2/3} + y^{2/3} = a^{2/3}$ where 'a' is the parameter. [8+8]

2. Solve $(D^3 - 3D^2 + 4D - 2)\, y = e^x + \cos x + x$. [16]

3. (a) Find the point on the plane x + 2y + 3z = 4 that is closest to the origin.
   (b) If f(x) = log x and g(x) = x^2 in [a, b], with b> a > 1, using Cauchy's Theorem prove that $\frac{\log b - \log a}{b-a} = \frac{a+b}{2c^2}$. [8+8]

4. (a) Find the radius of curvature of $\sqrt{a} = \sqrt{r}\cos\frac{\theta}{2}$ $at(r, \theta)$.
   (b) Find the envelope of the straight line $\frac{x}{a} + \frac{y}{b} = 1$ where $a^2 + b^2 = 4$. [8+8]

5. (a) Evaluate $\iint (x+y)\, dxdy$, over the region in the positive quadrant bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
   (b) Prove that the curved surface area of a sphere of radius r intercepted between two parallel planes at a distance a and b from the centre of the sphere is $2\pi r$ (b - a) when b > a and hence deduce the surface area of the sphere. [8+8]

6. (a) Examine the convergence of
   $\frac{3}{4} - \frac{5}{7} + \frac{7}{10} - \frac{9}{13} + \ldots$
   (b) Examine the convergence of
   $\frac{1}{1^2} + \frac{1}{3^2} - \frac{1}{5^2} - \frac{1}{7^2} + \ldots$ [6+10]

7. (a) Find the angle of intersection of the spheres $x^2 + y^2 + z^2 = 39$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z + 52 = 0$ at the point (4,-3,2).
   (b) Evaluate $\iint_S \vec{F} \cdot \vec{n}\, ds$ where $\vec{F} = 12x^2y\,\vec{i} - 3yz\vec{j} + 2z\,\vec{k}$ and S is the portion of the plane $x + y + z = 1$ included in the first octant. [8+8]

8. (a) Find L [f (t)] where f (t) is given by
$$f(t) = t, \quad 0 < t < b$$
$$= 2b - t, \; b < t < 2b,$$
2b being the period of f (t).

(b) Find L^{-1} [(2s +3)/ (s^3 - 6s^2 +11s - 6)] [8+8]

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Set No. 4

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Solve (x e^{xy} + 2y) $\frac{dy}{dx}$ + y e^{xy} = 0.
   (b) An object cools from 120^0 to 95^0F in half an hour when surrounded by air whose temperature is 70^0 F. Find its temperature at the end of another half an hour. [8+8]

2. Solve $\frac{d^4y}{dx^4}$ - y = cosx coshx + 2x^4 + x - 1. [16]

3. (a) Verify Rolle's theorem for f(x) = $x^{\frac{2}{3}} - 2x^{\frac{1}{3}}$ in the interval (0, 8).
   (b) If u = $\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$
   prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. [8+8]

4. (a) Find the radius of curvature of x = log t, y =$\frac{1}{2}(t + t^{-1})$ at t = 1.
   (b) Find the envelope of $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ where 'θ' is a parameter. [8+8]

5. (a) Evaluate $\int\limits_0^{\log 2}\int\limits_0^{x}\int\limits_0^{x+y} e^{x+y+z}dzdydx$.
   (b) Evaluate $\int\limits_0^{a}\int\limits_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - x^2 - y^2}\,dydx$. [8+8]

6. (a) Examine the convergence of
   $\sum$ (- 1)^n / (n + 1) log (n + 1)
   (b) Examine the convergence of
   x - x^2/2 + x^3/3 - x^4/4 + ... , (x > 0) [8+8]

7. Verify Gauss divergence theorem for $\vec{F} = x^2\,\vec{i} + y^2\vec{j} + z^2\,\vec{k}$, over the cube formed by the planes $x = 0, x = a, y = 0, y = b, z = 0, z = c$. [16]

8. (a) Find L^{-1} [1/ (s^4 +a^4)]

(b) Find $L^{-1}\left[1/ (s (s^2 +1) (s^2 +4) (s^2 +16)) \right]$. [8+8]

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