Code No: R059210101 **Set No. 1**

II B.Tech I Semester Regular Examinations, November 2006
MATHEMATICS-II
(Common to Civil Engineering, Mechanical Engineering, Chemical Engineering, Mechatronics, Metallurgy & Material Technology, Production Engineering, Aeronautical Engineering and Automobile Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Reduce the matrix A to its normal form. Where $A = \begin{pmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{pmatrix}$ and hence find the rank

   (b) For what values of K the system of equations will have a non-trivial solution and solve them for those values of K.
   2x + 3ky + (3k + 4)z = 0
   x + (k + 4)y + (4k + 2) z = 0
   x + 2(k + 1)y + (3k + 4)z = 0. [8+8]

2. (a) Define eigen value and eigen vector of a matrix A. Show that trace of A equals to the sum of the eigen values of A.

   (b) Verify that the sum of eigen values is equal to the trace of A for the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and find the corresponding eigen vectors. [8+8]

3. (a) Determine the index, signature and nature of the quadratic from:
   $4x^2$ + y^2 - $8z^2$ + 4xy - 4xz + 8yz.

   (b) Prove that the matrix $\begin{bmatrix} 0 & 1+i & 2-3i \\ -1+i & 4i & 4+5i \\ -2-3i & -4+5i & -3i \end{bmatrix}$ is skew Hermitian.[10+6]

4. (a) Find the Fourier series for f(x) ; if f(x) is defined in $-\pi <$ x $< \pi$ as
   $\mathrm{f}(\mathrm{x}) = \begin{cases} -\pi, -\pi < \mathrm{x} < 0 \\ \mathrm{x}, 0 < \mathrm{x} < \pi \end{cases}$

   Deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots\ldots\ldots$

   (b) Find the half range cosine series $\mathrm{f}(\mathrm{x}) = \mathrm{x}(2-\mathrm{x}), \mathrm{in} 0 \leq \mathrm{x} \leq 2$ and hence find the sum of series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots\ldots$ [10+6]

5. (a) Form the partial differential equation by eliminating the arbitrary function from z = xy + f (x^2 + y^2).

   (b) Solve the partial differential equation $2x^4p^2 - yzq = 3z^2$.

   (c) Solve the partial differential equation x (y - z) p + y (z - x) q = z (x - y). [5+5+6]

6. The temperature at one end of a bar is 50 cm long with insulated sides is kept at 0^o c and that the other end is kept at 100^o c until steady state condition prevails. The two ends are then suddenly insulated so that the temperature gradient is zero at each end thereafter. Find the temperature distribution. [16]

7. (a) Find the Fourier cosine transforms of e^{-ax} cos ax.

   (b) Prove that the Fourier transform of the convolution of f(x) and g(x) is the product of their Fourier transforms. [8+8]

8. (a) Find the Z transform of sin (3n+5)

   (b) Find $Z^{-1}\left[\frac{z}{z^2+11z+24}\right]$. [8+8]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find the rank of the matrix by reducing it to the normal form $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

   (b) Find whether the following system of equations
   x + 2y + z = 2
   3x + y - 2z = 1
   4x - 3y - z = 3
   2x + 4y + 2z = 4. [8+8]

2. (a) Find the eigen values and the corresponding eigen vectors of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

   (b) Prove that the product of the eigen values is equal to the determinent of the matrix. [10+6]

3. (a) Identify the nature, index, signature of the quadratic form $2x_1x_2 + 2x_2x_3 + 2x_3x_1$.

   (b) Prove that the matrix $\frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is unitary

   (c) Prove that transpose of a unitary matrix is unitary. [5+5+6]

4. (a) Express $\mathrm{f}(\mathrm{x}) = |\mathrm{x}|, -\pi < \mathrm{x} < \pi$ as Fourier series. Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2}........ = \frac{\pi^2}{8}$

   (b) $\mathrm{f}(\mathrm{x}) = \begin{cases} \frac{\pi}{3}, \mathrm{for} 0 \le \mathrm{x} < \frac{\pi}{3} \\ 0, \mathrm{for} \frac{\pi}{3} \le \mathrm{x} < \frac{2\pi}{3} \\ \frac{-\pi}{3} \mathrm{for} \frac{2\pi}{3} \le \mathrm{x} < \pi \end{cases}$ show that $\mathrm{f}(\mathrm{x}) = \frac{2}{\sqrt{3}}\left[\mathrm{cosx} - \frac{\mathrm{cos5x}}{5} + \frac{\mathrm{cos7x}}{7} - ....\right]$.

   [8+8]

5. (a) Form the partial differential equation by eliminating the arbitrary constants a, b from $2\mathrm{z} = (\mathrm{x}+\mathrm{a})^{1/2} + (\mathrm{y}-\mathrm{a})^{1/2} + \mathrm{b}$.

   (b) Solve the partial differential equation $(y^2 + z^2 - x^2)$ p ?2xyq +2xz = 0.

   (c) Solve the partial differential equation $p^2 + q^2 = x^2 + y^2$. [5+6+5]

6. The ends A and B of a rod 20 cm long have the temperature at 30^o c and 80^o c until study states prevail. The temperatures of the ends are changed at 40^o c and 60^o c respectively. Find the temperature distribution in the rod at time t. [16]

7. (a) State and prove Fourier Integral Theorem.

   (b) Find the Fourier transform of $f(x) = \begin{cases} e^{ikx} & a < x < b \\ 0 & x < a \text{ and } x > b \end{cases}$ [8+8]

8. (a) If $z[n] = \frac{z}{(z-1)^2}$, find z[n + 2]

   (b) Solve the difference equation, using Z - transforms
   y_{n+2} - $4y_{n+1} + 3y_n = 0$ given that $y_0 = 2$ and $y_1 = 4$. [8+8]

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Code No: R059210101 **Set No. 3**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find the rank of the matrix by reducing it to the normal form.
$$\begin{bmatrix} 4 & 3 & 2 & 1 \\ 5 & 1 & -1 & 2 \\ 0 & 1 & 2 & 3 \\ 1 & -1 & 3 & -2 \end{bmatrix}$$
   (b) Find whether the following set of equations is consistent if so solve them.
   2x - y + z = 5
   3x + y- 2z = - 2
   x - 3y - z = 2. [8+8]

2. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ verify cayley Hamilton theorem. Find A^4 and A^{-1} using cayley Hamilton theorem. [16]

3. Reduce the quadratic form $x^2 + y^2 + 2z^2$ - 2xy + 4xz + 4yz to the canonical form. [16]

4. (a) If f (x) = $\begin{cases} \sin x, \text{for} 0 \le \text{x} \le \pi \\ 0, \text{for} \pi \le \text{x} \le 2\pi \end{cases}$, obtain a Fourier series for f(x) of periodicity 2π and hence evaluate $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \ldots\ldots$
   (b) If f (x) = $\begin{cases} \sin x, \text{for} 0 \le \text{x} \le \frac{\pi}{2} \\ \cos x, \text{for} \frac{\pi}{4} \le \text{x} \le \pi \end{cases}$ expand f(x) in a series of half range sines. [10+6]

5. (a) Form the partial differential equations by eliminating the arbitrary functions z= y^2 + 2f(1/x +log y)
   (b) Solve the partial differential equation (x^2 - y^2 - z^2)p + 2xyq = 2xz.
   (c) Solve the partial differential equation z^2 (p^2 x^2 + q^2) =1. [5+6+5]

6. (a) $4u_x + u_y$ = 3u given u = $3e^{-y}$ - e^{-5y} when x = 0.
   (b) Find the general solution of one-dimensional heat equation. [8+8]

7. (a) State and prove Fourier Integral Theorem.
   (b) Find the Fourier transform of $f(x) = \begin{cases} e^{ikx} & a < x < b \\ 0 & x < a \text{ and } x > b \end{cases}$ [8+8]

8. (a) If $z[n] = \frac{z}{(z-1)^2}$, find z[n + 2]

   (b) Solve the difference equation, using Z - transforms
   y_{n+2} - $4y_{n+1} + 3y_n = 0$ given that $y_0 = 2$ and $y_1 = 4$. [8+8]

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Code No: R059210101 **Set No. 4**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Find the rank of the matrix by reducing it to the normal form
$$\begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$$
   (b) Find whether the following set of equations are consistent if so, solve them.
   3x + y + z = 8, -x + y - 2z = - 5
   2x + 2y + 2z = 12, -2x + 2y - 3z = -7. [8+8]

2. Verify Cayley Hamilton theorem and hence evaluate $A^{-1}, A = \begin{bmatrix} 1 & 3 & 7 \\ 1 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ [16]

3. (a) Show that $A = \begin{bmatrix} a+ic & -b+id \\ b+id & a-ic \end{bmatrix}$ is unitary matrix if $a^2 + b^2 + c^2 + d^2 = 1$.
   (b) Find the eigen values and the corresponding eigen vector of $\begin{bmatrix} 4 & 1-3i \\ 1+3i & 7 \end{bmatrix}$ [8+8]

4. (a) Expand f(x) = cos ax as a Fourier series in (-π, π) where a is not an integer. Hence prove that $\cot\theta = \frac{1}{\theta} + \frac{2\theta}{\theta^2-\pi^2} + \frac{2\theta}{\theta^2-4\pi^2} + ...$
   (b) If $f(x) = x, 0 < x < \frac{\pi}{2}$
   $= \pi - x, \frac{\pi}{2} < x < \pi$
   Show that $f(x) = \frac{4}{\pi}\left[\sin x - \frac{1}{3^2}\sin 3x + \frac{1}{5^2}\sin 5x - ....\right]$. [8+8]

5. (a) Form the partial differential equation by eliminating the arbitrary constants a, b from $2z = (x + a)^{1/2} + (y - a)^{1/2} + b$.
   (b) Solve the partial differential equation (y^2 + z^2 - x^2) p ?2xyq +2xz = 0.
   (c) Solve the partial differential equation $p^2 + q^2 = x^2 + y^2$. [5+6+5]

6. Find the study state temperature in a rectangular plate $0 < x < a, 0 < y < b$ when the sides x = 0, x= a, y = b are insulated while the edge y = 0 is kept at temperature k cos (Πx/a). [16]

7. (a) Find Fourier cosine transform of $e^{-a^2x^2}$ and hence find sine transform of $xe^{-a^2x^2}$

(b) Find the finite sine and cosine transform of $x^3 \quad 0 < x < \pi$. [10+6]

8. (a) Z - transform of n cos n θ.

   (b) Find $Z^{-1}\left[\frac{1}{(z-5)^3}\right]$ |z| > 5. Determine the region of convergence. [6+10]

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