Code No: R059210101 **Set No. 1**

II B.Tech I Semester Supplimentary Examinations, February 2008
MATHEMATICS-II
(Common to Civil Engineering, Mechanical Engineering, Chemical Engineering, Mechatronics, Metallurgy & Material Technology, Production Engineering, Aeronautical Engineering and Automobile Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Find the non singular matrices P and Q such that PAQ is in the normal form of the matrix and find the rank of the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{pmatrix}$

   (b) Find the values of a and b for which the equations
   x + ay + z = 3
   x + 2y + 2z = b
   x + 5y + 3z = 9 will have
      i. Unique solution
      ii. No solution
      iii. Infinite no of solutions. [8+8]

2. (a) Show that if λ_1, λ_2 ??. λ_n are latent roots of a matrix A, then A^3 has the latent roots λ_1^3, λ_2^3, ...λ_n^3 and kλ_1, kλ_2, kλ_3.............. kλ_n are latent roots of kA.

   (b) Find the eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ [6+10]

3. (a) Define the following:
      i. Hermitian matrix
      ii. Skew-Hermitain matrix
      iii. Unitary matrix
      iv. Orthogonal matrix.

   (b) Show that the eigen values of an unitary matrix is of unit modulus. [8+8]

4. (a) Express $\mathrm{f}(\mathrm{x}) = |\mathrm{x}|, -\pi < \mathrm{x} < \pi$ as Fourier series. Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2}........ = \frac{\pi^2}{8}$

   (b) $\mathrm{f}(\mathrm{x}) = \begin{cases} \frac{\pi}{3}, \mathrm{for} 0 \leq \mathrm{x} < \frac{\pi}{3} \\ 0, \mathrm{for} \frac{\pi}{3} \leq \mathrm{x} < \frac{2\pi}{3} \\ \frac{-\pi}{3} \mathrm{for} \frac{2\pi}{3} \leq \mathrm{x} < \pi \end{cases}$ show that $\mathrm{f}(\mathrm{x}) = \frac{2}{\sqrt{3}} \left[\mathrm{cosx} - \frac{\mathrm{cos5x}}{5} + \frac{\mathrm{cos7x}}{7} - ....\right]$.

   [8+8]

5. (a) Form the partial differential equation by eliminating the arbitrary constants from $(x-a)^2 + (y-b)^2 + z^2 = r^2$.
   (b) Solve the partial differential equation x(y-z)p + y(z-x)q = z(x-y).
   (c) Solve the partial differential equation y^2 z p + x^2 z q = xy^2. [5+6+5]

6. Solve the problem of the vibrating string with the boundary conditions (with usual notations) u(0, t) = 0 = u(L, t) and initial conditions u t (x, o) = 0 and
u(x, 0) = $x, 0 < x < \frac{L}{2}$
$= (L-x)\,, \frac{L}{2} < x < L$ [16]

7. (a) Find the finite Fourier cosine transform of f(x) = x^2 in (0, l).
   (b) Find the Fourier cosine transform of $5e^{-2x} + 2e^{-5x}$. [8+8]

8. (a) If Z(u_n) =$\overline{u}$(z) and k > 0 then prove that
      i. Z(u_{n-k}) = $z^{-k}\,\overline{u}$(z)
      ii. Z(u_{n+k}) = $z^k \left[\overline{u}(z)\ - u_o - u_1 z^{-1} u_2 \overline{z}^{-2} ... u_{k-1} z^{-(k-1)}\right]$. [5+5]
   (b) Using convolution theorem evaluate $Z^{-1}\left[\frac{z^2}{(z-1)\,(z-3)}\right]$ [6]

⋆⋆⋆⋆⋆

Code No: R059210101 **Set No. 2**

II B.Tech I Semester Supplimentary Examinations, February 2008
MATHEMATICS-II
(Common to Civil Engineering, Mechanical Engineering, Chemical Engineering, Mechatronics, Metallurgy & Material Technology, Production Engineering, Aeronautical Engineering and Automobile Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Find the value of λ for which the system of equations
3x - y + 4z = 3, x + 2y - 3z = - 2
6x + 5y + λz = - 3
will have infinite no of solutions and solve them with that λ value.
(b) Find the rank of the matrix A by reducing it to the normal form Where
$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & -4 \\ 2 & 3 & 5 & -5 \\ 3 & -4 & -5 & 8 \end{bmatrix}.$$ [8+8]

2. (a) Find the eigen values and the corresponding eigen vectors of the matrix
$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$
(b) If $\lambda_1, \lambda_2 ... \lambda_n$ are eigen values of A, then prove that $\lambda_1^m, \lambda_2^m ... \lambda_n^m$ are eigen values of A^m. [10+6]

3. (a) Show that the eigen value of an orthogonal matrix is of unit modulus.
(b) Show that is a Hermitian matrix $\begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$
(c) Prove that the matrix $\begin{bmatrix} \frac{-2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-2}{3} & \frac{2}{3} \end{bmatrix}$ is orthogonal. [5+6+5]

4. (a) Find the Fourier series to represent f(x) = x^2 - 2, when $-2 \le$ x ≤ 2
(b) Obtain a half range cosine series for f (x) = $\begin{cases} \text{kx}, 0 \le \text{x} \le \frac{\text{L}}{2} \\ \text{k (L} - \text{x)}, \frac{\text{L}}{2} \le \text{x} \le \text{L} \end{cases}$
Deduce the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2}$........ [10+6]

5. (a) Form the partial differential equation by eliminating the arbitrary constants a, b from 2z = $(\text{x} + \text{a})^{1/2}$+$(\text{y} - \text{a})^{1/2}$+b.
(b) Solve the partial differential equation (y^2 + z^2 - x^2) p ?2xyq +2xz = 0.
(c) Solve the partial differential equation $p^2 + q^2 = x^2 + y^2$. [5+6+5]

6. An infinitely long plate is bounded by two parallel edges and an end at right angles to them. The breadth is π. This end is maintained at constant temperature u_0 at all points and the other edges are at zero temperature. Find the steady state temperature at any point (x,y) of the plate. [16]

7. (a) Find the finite Fourier sine and cosine transforms of
   i. f (x) = x in (0, l).
   (b) Find the finite sine and transform of f(x) = cos kx in $0 < x < \pi$ [8+8]

8. (a) If $z\,[n] = \frac{z}{(z-1)^2}$, find z[n + 2]
   (b) Solve the difference equation, using Z - transforms
   y_{n+2} - $4y_{n+1}$ + $3y_n = 0$ given that $y_0 = 2$ and $y_1 = 4$. [8+8]

⋆⋆⋆⋆⋆

Code No: R059210101 **Set No. 3**

II B.Tech I Semester Supplimentary Examinations, February 2008
MATHEMATICS-II
(Common to Civil Engineering, Mechanical Engineering, Chemical Engineering, Mechatronics, Metallurgy & Material Technology, Production Engineering, Aeronautical Engineering and Automobile Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Determine the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 5 & 4 \\ 1 & 0 & 2 & -3 & 5 \\ -1 & 2 & 3 & 4 & -2 \end{bmatrix}$ by reducing it to the normal form.

   (b) Find whether the following set of equations is consistent if so solve them
   x + y + z = 8
   2x + 3y + 2z = 19
   4x + 2y + 3z = 23. [8+8]

2. (a) Define eigen value and eigen vector of a matrix A. Show that trace of A equals to the sum of the eigen values of A.

   (b) Verify that the sum of eigen values is equal to the trace of A for the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and find the corresponding eigen vectors. [8+8]

3. (a) Show that the eigen value of an orthogonal matrix is of unit modulus.

   (b) Show that is a Hermitian matrix $\begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$

   (c) Prove that the matrix $\begin{bmatrix} \frac{-2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-2}{3} & \frac{2}{3} \end{bmatrix}$ is orthogonal. [5+6+5]

4. (a) Find a Fourier series to represent x − x^2 from x = $-\pi$ to x =π.. Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ...$

   (b) Find the half range sine series for the function
   $f(x) = \begin{cases} \frac{1}{4} - x, 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, \frac{1}{2} < x < 1 \end{cases}$ [10+6]

5. (a) Form the partial differential equations by eliminating the arbitrary functions z= y^2 + 2f(1/x +log y)

   (b) Solve the partial differential equation (x^2 - y^2 - z^2)p + 2xyq = 2xz.

   (c) Solve the partial differential equation z^2 (p^2 x^2 + q^2) =1. [5+6+5]

6. (a) Solve by separation of variables $3u_x + 2u_y = 0$ with u(x,0) = 4 e^{-x}.
   (b) Obtain the general solution of the one dimension wave equation $\partial^2 u/\partial t^2 = c^2\, \partial 2u\,/\partial x^2$. [8+8]

7. (a) Find the finite Fourier cosine transform of f(x) = x^2 in (0, l).
   (b) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$ (a > 0) and deduce that $\int\limits_0^\infty \frac{e^{-ax} - e^{-bx}}{x}$ sinsx dx = $\tan^{-1}\left(\frac{b}{s}\right) - \tan^{-1}\left(\frac{a}{s}\right)$. [8+8]

8. (a) State and Prove damping rule.
   (b) Find Z (cos h at. sin bt)
   (c) Find the inverse Z transform of $\frac{z}{z^2+7z+10}$. [5+6+5]

⋆ ⋆ ⋆ ⋆ ⋆

Code No: R059210101 **Set No. 4**

II B.Tech I Semester Supplimentary Examinations, February 2008
MATHEMATICS-II
(Common to Civil Engineering, Mechanical Engineering, Chemical Engineering, Mechatronics, Metallurgy & Material Technology, Production Engineering, Aeronautical Engineering and Automobile Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Find the rank of the matrix $A = \begin{pmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{pmatrix}$ by reducing it to the normal form.
   (b) Find whether the following equations are consistent, if so solve them.
   $x_1 + 2x_2 + 3x_3 = 16$
   $x_1 + x_2$ - $3x_3$ = -9
   x_1 - $2x_2 + 2x_3 = 8$. [8+8]
2. (a) Show that if λ_1, λ_2 ??. λ_n are latent roots of a matrix A, then A^3 has the latent roots λ_1^3, λ_2^3, ...λ_n^3 and kλ_1, kλ_2, kλ_3............... kλ_n are latent roots of kA.
   (b) Find the eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ [6+10]
3. (a) Prove that the eigen values of a real symmetric matrix are real.
   (b) Reduce the quadtatic form $7x^2 + 6y^2 + 5z^2$ - 4xy - 4yz to the canonical form. [6+10]
4. (a) Obtain a Fourier expansion for $\sqrt{1 - \cos x}$ in the interval $-\pi<$ x $<\pi$
   (b) Represent the following function by a Fourier sin series f (t) = $\begin{cases} t, 0 < t \leq \frac{\pi}{2} \\ \frac{\pi}{2}, \frac{\pi}{2} < t \leq \pi \end{cases}$ [10+6]
5. (a) Form the partial differential equation by eliminating the arbitrary function from $f\left(x^2 + y^2, z - xy\right) = 0$.
   (b) Solve the partial differential equation $(2z - y)\,p + (x + z)\,q + 2x + y = 0$.
   (c) Solve the partial differential equation $z^4P^2 + z^4q^2 = z^3$. [5+6+5]
6. (a) Solve the following equation by the method of separation of variables $z_{xx} - 2z_x + zy = 0$.
   (b) Find the general solution of Laplace equation $\partial^2u/\partial x^2 + \partial^2u/\partial y^2 = 0$. [8+8]

7. (a) State and prove Fourier Integral Theorem.

   (b) Find the Fourier transform of $f(x) = \begin{cases} e^{ikx} & a < x < b \\ 0 & x < a \text{ and } x > b \end{cases}$ [8+8]

8. (a) Find the Z - Transform of $a^n \cos n\theta$

   (b) Using Z - transforms, solve the difference equation: $y_{n+2} + y_n = 2$ given that $y_0 = y_1 = 0$. [6+10]

⋆⋆⋆⋆⋆