Code No: R05010102 **Set No. 1**

I B.Tech Regular Examinations, Apr/May 2007
MATHEMATICS-I
(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering and Automobile Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Test the convergence of the series $\frac{2}{1} + \frac{2.5.8}{1.5.9} + \frac{2.5.8.11}{1.5-9.13} + ... \infty$. [5]

   (b) Find whether the following series converges absolutely / condtionally $\frac{1}{6} - \frac{1}{6}.\frac{1}{3} + \frac{1.3.5}{6.8.10} \frac{-1.3.5.7}{6.8.10.12}$. [5]

   (c) Prove that $\frac{\pi}{6} + \frac{\sqrt{3}}{5} < \sin^{-1}\frac{3}{5} < \pi/6 + \frac{1}{8}$. [6]

2. (a) Show that the functions u = x+y+z , v = x^2+y^2+z^2-2xy-2zx-2yz and w = x^3+y^3+z^3-3xyz are functionally related. Find the relation between them.

   (b) Find the centre of curvature at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$ of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$. Find also the equation of the circle of curvature at that point. [8+8]

3. (a) Find the length of the curve $x^2(a^2 - x^2) = 8\ a^2y^2$.

   (b) Find the volume of the solid generated by revolving the lemniscates $r^2 = a^2$ Cos 2θ about the line $\theta = \frac{\pi}{2}$. [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant : log y/x = cx. [3]

   (b) Solve the differential equation: $(1+ y^2)\ dx = (\tan^{-1}y - x)\ dy$. [7]

   (c) The temperature of the body drops from 100^0 C to 75^0C in ten minutes when the surrounding air is at 20^0C temperature. What will be its temperature after half an hour. When will the temperature be 25^0C. [6]

5. (a) Solve the differential equation: $(D^2$-1)y= xsinx + $x^2\ e^x$.

   (b) Solve the differential equation: $(x^2D^2$+xD+4)y=log x cos (2logx). [8+8]

6. (a) Prove that L $\left[\left[\frac{1}{t} f(t)\right]\right] = \int_s^\infty \overline{f}(s)\, ds$ where L [f(t)] = $\overline{f}\,(s)$ [5]

   (b) Find the inverse Laplace Transformation of $\frac{3(s^2-2)^2}{2\, s^5}$ [6]

(c) Evaluate $\int\int (x^2 + y^2)dxdy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ [5]

7. (a) For any vector **A,** find div curl **A.** [6]

   (b) Evaluate $\iint_s$ **A**.**n** ds where **A**=z **i** +x **j**-3y^{2}z **k** and S is the surface of the cylinder $x^2 + y^2 = 16$included in the first octant between z=0 and z=5. [10]

8. Verify Stoke's theorem for **F** = -y^3**i**+x^3**j** in the region $x^2+y^2 \leq 1$, z=0. [16]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Test the convergence of the following series $\sum\left(\frac{n^2}{2^n} + \frac{1}{n^2}\right)$ [5]
   (b) Find the interval of convergence of the series whose n th term is $\sum\frac{(-1)^n(n+2)}{(2^n+5)}$ [5]
   (c) If a < b prove that $\frac{b-a}{(1+b^2)} < tan^{-1}b - tan^{-1}a < \frac{b-a}{(1+a^2)}$ using Lagrange's Mean value theorem. Deduce the following [6]
      i. $\frac{\Pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\Pi}{4} + \frac{1}{6}$
      ii. $\frac{5\Pi+4}{20} < \tan^{-1}2 < \frac{\Pi+2}{4}$
2. (a) If u=x^2-y^2, v=2xy where x=r cosθ, y=rsinθ. Show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 4r^3$.
   (b) For the cardioid r=a(1+cosθ) Prove that $\frac{\rho^2}{r}$ is constant where rho is the radius of curvature. [8+8]
3. (a) Find the volume of the solid generated by revolution of $y^2 = \frac{x^3}{(2a-x)}$ about its asymptote.
   (b) Find the area of the loop of the curve r=a(1+cos θ). [8+8]
4. (a) Form the differential equation by eliminating the arbitrary constant y = $\frac{a+x}{x^2+1}$. [3]
   (b) Solve the differential equation: (1-x ^{2})$\frac{dy}{dx}$ - xy = y^{3}sin $^{-1}$x. [7]
   (c) Prove that the family of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ are self orthogonal (λ the parameter) [6]
5. (a) Solve the differential equation: $(D^3 - 7D^2 + 14D - 8)y = e^x cos2x$.
   (b) Solve the differential equation: $(x^2D^2 - x3D + 1)y = \frac{\log x \sin(\log x)+1}{x}$. [8+8]
6. (a) Solve the differential equation $\frac{d^2x}{dx^2} + 9x = Sin\,t$ using Laplace transforms given that x(0) = 1, x($\pi/2$) =1

(b) Change the order of integration hence evaluate $\int_0^1 \int_{x^2}^{2-x} xdy\,dx$ [8+8]

7. (a) Prove that ∇x(∇x**A**) = -∇^2**A**+∇(∇.**A**).

   (b) If $\phi = 2xy^2z + x^2y$, evaluate $\int_C \phi$ d**r** where C consists of the straight lines from (0, 0, 0) to (1, 0, 0) then to (1, 1, 0) and then to (1, 1, 1). [8+8]

8. Verify Green's theorem for $\oint_C (y - Sin\,x\,)\,dx + Cos\,x\,dy$ where C is the triangle formed by the points (0,0) (π/2, 0) and (π/2, 1). [16]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
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1. (a) Test the convergence of the following series $\sum \frac{1}{(\log \ \log n)^n}$ [5]
   (b) Find the interval of convergence of the series $x + \frac{1}{2}.\frac{x^3}{3} + \frac{1}{2}.\frac{3}{4}.\frac{x^5}{5} + \frac{1.3.5}{2.4.6}.\frac{x^7}{7} + ...$ [5]
   (c) Show that $\log\,(1+e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + ...$ and hence deduce that $\frac{e^x}{e^x+1} = \frac{1}{2} + \frac{x}{4} - \frac{x^3}{48} + ...$ [6]

2. (a) Given that x+y+z=a, find the maximum value of $x^m y^n z^p$.
   (b) Find the envelope of the circles through the origin and whose centre lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2}$. [8+8]

3. (a) Trace the curve : r = a (1 + cos θ).
   (b) Find the length of the arc of the curve x = $e^\theta \sin\theta$; y = $e^\theta \cos\theta$ from $\theta = 0$ to $\theta = \pi/2$. [8+8]

4. (a) Find the differential equation of all parabolas having the axis as the axis and (a,0) as the focus.
   (b) Solve the differential equation $\frac{x^2 dy}{dx} = e^y - x$.
   (c) Find the orthogonal trajection of the family of curves $x^{2/3} + y^{2/3} = a^{2/3}$. [4+6+6]

5. (a) Solve the differential equation: $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -2\,Coshx$ given that y(0) = 0, y′(0) = 1.
   (b) Solve the differential equation: $(2x-1)^3\frac{d^3y}{dx^3} + (2x-1)\frac{dy}{dx} - 2y = x.$ [8+8]

6. (a) Evaluate L{e^t(cos2t + ½ sinh2t)} [5]
   (b) Find $L^{-1}\left[\frac{1}{s^2+2s+5}\right]$ [6]
   (c) Evaluate the triple integral $\int_0^1\int_y^1\int_0^{1-x} x\,dz\,dx\,dy$ [5]

7. (a) Evaluate $\nabla^2 \log r$ where $r = \sqrt{x^2 + y^2 + z^2}$
   (b) Find constants a, b, c so that the vector $\mathbf{A}$ =(x+2y+az)$\mathbf{i}$ +(bx-3y-z)$\mathbf{j}$+(4x+cy+2z)$\mathbf{k}$ is irrotational. Also find φ such that $\mathbf{A} = \nabla\phi$. [8+8]

8. Verify Stoke's theorem for the vector field $\mathbf{F}$=(2x-y)$\mathbf{i}$-yz^2$\mathbf{j}$-y^{2}z$\mathbf{k}$ over the upper half surface of x^2+y^2+z^2=1, bounded by the projection of the xy-plane. [16]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Test the convergence of the series $\frac{\sqrt{2}-1}{3^2-1} + \frac{\sqrt{3}-1}{4^2-1} + \frac{\sqrt{4}-1}{5^2-1} + ...$ [5]
   (b) Examine whether the following series is absolutely convergent or conditionally convergent $1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \ldots\ldots$ [5]
   (c) Verify Rolle's theorem for f(x) = $\log\left[\frac{x^2+ab}{x(a+b)}\right]$ in [a,b] (x $\neq$ 0). [6]

2. (a) Show that the functions u = x+y+z , v = $x^2+y^2+z^2$-2xy-2zx-2yz and w = $x^3+y^3+z^3$-3xyz are functionally related. Find the relation between them.
   (b) Find the centre of curvature at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$ of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$. Find also the equation of the circle of curvature at that point. [8+8]

3. (a) In the evolute of the parabola y^2= 4ax, show that the length of the curve from its cusp x = 2a to the point where it meets the parabola y^2 = 4ax is 2a($3\sqrt{3}$ - 1)
   (b) Find the length of the arc of the curve $y = \log\left[\frac{e^x-1}{e^x+1}\right]$from x = 1 to x = 2 [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant : log y/x = cx. [3]
   (b) Solve the differential equation: (1+ y^2) dx = ($\tan^{-1}$y – x) dy. [7]
   (c) The temperature of the body drops from 100^0 C to 75^0C in ten minutes when the surrounding air is at 20^0C temperature. What will be its temperature after half an hour. When will the temperature be 25^0C. [6]

5. (a) Solve the differential equation: $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = Sin\,2x$.
   (b) Solve the differential equation: $x^2\frac{d^2y}{dx^2} - 2x\,\frac{dy}{dx} - 4y = x^4$. [8+8]

6. (a) Find L [t^2 Sin2t] [5]
   (b) Find $L^{-1}\left[\frac{s+3}{(s^2-10s+29)}\right]$ [6]

(c) Evaluate $\int_0^{\pi/4} \int_0^{a\,Sin\theta} \frac{r\,dr\,d\theta}{\sqrt{a^2 - r^2}}$ [5]

7. (a) Prove that $\nabla \times \left(\frac{\bar{A}\times\bar{r}}{r^n}\right) = \frac{(2-n)\bar{A}}{r^n} + \frac{n\left(\bar{r}.\bar{A}\right)\bar{r}}{r^{n+2}}$

   (b) If $\bar{F} = (x^2 - 27)\,i - 6yzj + 8xz^{2k}$ evaluate $\int_C \bar{F}.d\bar{r}$ from the point (0,0,0) to the point (1,1,1) along the straight line from (0,0,0) to (1,0,1), (1,0,0) to (1,1,0) and (1,1,0) to (1,1,1) [8+8]

8. Verify Stokes theorem f=x^2i-yzj+k integrated around the square x=0, y=0, z=0, x=1, y=1 and z=1. [16]

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