Code No: R059010102 **Set No. 1**

I B.Tech Regular Examinations, May/Jun 2006
MATHEMATICS-I
(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering and Automobile Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the following series $\sum_{n=1}^{\infty} \frac{x^{2n}}{(n+1)\sqrt{n}}$. [5]
   (b) Find the interval of convergence of the series $\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ...\infty$. [5]
   (c) Write Taylor's series for f(x) = $(1 - x)^{5/2}$ with Lagrange's form of remainder upto 3 terms in the interval [0,1]. [6]

2. (a) If x+y+z=u, y+z=uv, z=uvw, then evaluate $\frac{\partial(x,y,z)}{\partial(u,v,w)}$
   (b) IF ρ_1 and ρ_2 are radii of curvatures of any chord of the cardioids r=a(1+cosθ) which passes through the pole, then show that $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$. [8+8]

3. (a) In the evolute of the parabola y^2= 4ax, show that the length of the curve from its cusp x = 2a to the point where it meets the parabola y^2 = 4ax is 2a(3$\sqrt{3}$ - 1)
   (b) Find the length of the arc of the curve $y = \log\left[\frac{e^x-1}{e^x+1}\right]$from x = 1 to x = 2 [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constants y=e^x(acosx+bsinx).
   (b) Solve the differential equation y(2xy+e^x)dx-e^xdy=0.
   (c) A body kept in air with temperature 25^oC cools from 140^C to 80^o in 20minutes. Find when the body cools down to0 35^oC. [4+6+6]

5. (a) Solve the differential equation: (D^2+2D-3) y=x^2e^{-3x}.
   (b) Solve the differential equation: (D^2+4)=sec2x by the method of variation of parameters. [8+8]

6. (a) If $[f(t)] = \frac{9s^2-12s+5}{(s-1)^3}$ find L [f(3t)] using change of scale property
   (b) Find $L^{-1}\left[\frac{s}{(s^2+4)^2}\right]$

(c) Evaluate $\iint (x^2 + y^2)\, dxdy$ in the positive quadrant for which $x + y \ \leq 1$ [5+5+6]

7. (a) Find the work done by $\overline{F} = (2x - y - z)\, i + (x + y - z)\, j + (3x - 2y - 5z)\ k$ along a curve C in the xy-plane given by $x^2 + y^2 = 9$, z = 0. [8+8]

   (b) IF the scalar fields u and v and a vector field F are such that $u\,\bar{F} = \nabla v$, then prove that curl $\bar{F} = 0$.

8. (a) Find the area of the Folium of Descartes x^3+y^3=3axy $(a > 0)$ using Green's theorem.

   (b) Use Divergence theorem to evaluate $\iint (xi + yi + z^2 k).\bar{n}ds$ where S is the surface bounded by the cone x^2+y^2=z^2 in the plane Z=4.

[8+8]

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1. (a) Test the convergence of the following series $\sum_{n=1}^{\infty} \frac{1.3.5.....(2n+1)}{2.5.8....(3n+2)}$ [5]

   (b) Prove that the series$\frac{(-1)^n}{n(logn)^3}$ converges absolutely. [5]

   (c) Show that $\frac{\sin^{-1} x}{\sqrt{1-x^2}} = x + \frac{x^3}{3!} + \frac{1^2.3^2}{5!} x^5 + \frac{1^2.3^2.5^2}{7!} x^7 + ...$ [6]

2. (a) If u=x^2-y^2, v=2xy where x=r cosθ, y=rsinθ. Show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 4r^3$.

   (b) For the cardioid r=a(1+cosθ) Prove that $\frac{\rho^2}{r}$ is constant where rho is the radius of curvature. [8+8]

3. (a) Trace the curve : $y^2 = (x -2) (x -3)^2$.

   (b) Prove that the volume of revolution of r^2=$a^2 cos2\ \theta$ about the initial line is $\frac{\pi a^3}{6\sqrt{2}} \left[3 \ \log(\sqrt{2} + 1) - \sqrt{2}\right]$ [16]

4. (a) Form the differential equation by eliminating the arbitrary constant y = 1 + c$\sqrt{1 - x^2}$. [3]

   (b) Solve the differential equation: (1+$e^{x/y}$) dx + (1 − x/y)$e^{x/y}$ dy = 0. [7]

   (c) In a certain chemical reaction the rate of conversion of a substance at time t is proportional to the quantity of the substance still untransformed at that instant. At the end of one hour 60 grams remain and at the end of four hours 21 grams. How many grams of the first substance was there initially? [6]

5. (a) Solve the differential equation: (D^2+1)y=$x^2 e^{3x}$.

   (b) Solve the differential equation: (x^2 D^2-4xD+6)y=$(logx)^2$. [8+8]

6. (a) Prove that L $\left[\left[\frac{1}{t} f(t)\right] = \int_s^{\infty} \overline{f}(s)\, ds\right.$ where L [f(t)] = $\overline{f}\,(s)$ [5]

   (b) Find the inverse Laplace Transformation of $\frac{3(s^2-2)^2}{2\,s^5}$ [6]

(c) Evaluate $\int\int (x^2 + y^2)dxdy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ [5]

7. (a) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2, -1, 2)

   (b) Evaluate $\nabla.(\mathbf{r}/r^3)$ where $\mathbf{r} = xi+yj+zk$ and $r = |\mathbf{r}|$ [8+8]

8. State Green's theorem and verify Green's theorem for $\oint_C [(xy + y^2)dx + x^2dy]$, where C is bounded by $y = x$ and $y = x^2$. [16]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Test the convergence of the following series $\sum \frac{(2n+1)}{n^3+1} x^n$, $x > 0$. [5]

   (b) Test the following series for absolute /conditional convergence $\sum_{n=1}^{\infty} (-1)^n \frac{\log n}{n^2}$ [5]

   (c) Verify cauchy's mean value theorem for f(x) = e^x and g(x) = e^{-x} in [a,b] [6]

2. (a) Find the stationary points of u(x,y) = Sinx Siny Sin(x + y) where 0 < x < $\prod$, 0<y<$\prod$ and find the maximum u.

   (b) Find the radius of curvature of the curve $x^2y = a(x^2+y^2)$ at (-2a , 2a) [8+8]

3. (a) Trace the curve r = a + b Cos θ. (a > b).

   (b) Obtain the surface area of the solid of revolution of the curve r = a (1 + cosθ) about the initial line. [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant 'c':
   $y = 1 + x^2 + c\sqrt{1+x^2}$. [3]

   (b) Solve the differential equation:
   $\frac{dy}{dx} + (y - 1) \cos x = e^{-sinx}\cos^2 x$. [7]

   (c) The temperature of cup of coffee is 92^0 C, when freshly poured the room temperature being 24^0 C. In one minute is was cooled to 80^0 C. how long a period must elapse, before the temperature of the cup becomes 65^0 C. [6]

5. (a) Solve the differential equation: $(D^3 - 4D^2 - D + 4)y = e^{3x}$ cos 2x.

   (b) Solve the differential equation: $y''' + 2y'' - y' - 2y = 1 - 4x^3$. [8+8]

6. (a) If $[f(t)] = \frac{9s^2-12s+5}{(s-1)^3}$ find L [f(3t)] using change of scale property

   (b) Find $L^{-1}\left[\frac{s}{(s^2+4)^2}\right]$

(c) Evaluate $\iint (x^2 + y^2)\, dxdy$ in the positive quadrant for which $x + y \leq 1$ [5+5+6]

7. (a) If ω is constant vector, evaluate curl **V** where **V**=$\omega\times$**r**.

   (b) Evaluate $\int_c$ **F**.d**r** where **F**=(x-3y)**i**+(y-2x)**j** and c is the closed curve in the xy-plane, x=2cost,y=3sint from t=0 to t=2π. [8+8]

8. Verify divergence theorem for F = x^2 i + y^2 j + z^2 k over the surface S of the solid cut off by the plane x + y + z = a in the first octant. [16]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the following series $\sum \frac{1}{(\log\ \log n)^n}$ [5]
   (b) Find the interval of convergence of the series
   $x + \frac{1}{2}.\frac{x^3}{3} + \frac{1}{2}.\frac{3}{4}.\frac{x^5}{5} + \frac{1.3.5}{2.4.6}.\frac{x^7}{7} + ...$ [5]
   (c) Show that $\log(1+e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + ...$ and hence deduce that $\frac{e^x}{e^x+1} = \frac{1}{2} + \frac{x}{4} - \frac{x^3}{48} + ...$ [6]

2. (a) IF $x + y^2$=u, $y + z^2$=v, $z + x^2$=w find $\frac{\partial x}{\partial u}$
   (b) Prove that the evolute of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is .$(ax)^{2/3} - (by)^{2/3} = (a^2+b^2)^{2/3}$ by considering evolute as the envelope of the normal. [8+8]

3. (a) Trace the curve : x = a ($\theta-$ sin θ) ; y = a (1 - cos θ).
   (b) A sphere of radius 'a' units is divided into two parts by a plane distant (a/2) from the centre. Show that the ratio of the volumes of the two parts is 5:11. [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant y^2=$(x-c)^2$.
   (b) Solve the differential equation $\frac{y+x-2}{y-x-4} = \frac{dy}{dx}$.
   (c) If the temperature of the air is 20^oC and the temperature of the body drops from 100^oC to 80^oC in 10 minutes. What will be its temperature after 20 minutes when will be the temperature 40^oC. [3+7+6]

5. (a) Solve the differential equation $(D^2$+4)y=e^x+sin2x.
   (b) Solve the differential equation (x^2D^2-x D+1)y=logx. [8+8]

6. (a) Find $L^{-1}\left[\frac{s^2}{(s^4+4)(s^2+9)}\right]$ using convolution theorem.

(b) Show that $\int_{0}^{4a} \int_{y^2/4a}^{y} \frac{\left(x^2-y^2\right)dxdy}{(x^2+y^2)} = 8\,a^2\left(\frac{\pi}{2} - \frac{5}{3}\right)$ [8+8]

7. (a) Find curl$[\mathbf{r}f(r)]$ where $\mathbf{r}$ = x$\mathbf{i}$+y$\mathbf{j}$+z$\mathbf{k}$, r = $|\mathbf{r}|$. [6]

   (b) Find the work done in moving a particle in the force field $\mathbf{F}$=$3x^2i + j + zk$ along the straight line from (0, 0, 0) to (2, 1, 3). [10]

8. Verify divergence theorem for $2x^2y\mathbf{i} - y^2\mathbf{j} + 4xz^2\mathbf{k}$ taken over the region of first octant of the cylinder $y^2+z^2 = 9$ and x = 2. [16]

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