

I B.Tech Regular Examinations, May/June 2008**MATHEMATICS-I**

(Common to Civil Engineering, Electrical & Electronic Engineering,
 Mechanical Engineering, Electronics & Communication Engineering,
 Computer Science & Engineering, Chemical Engineering, Electronics &
 Instrumentation Engineering, Bio-Medical Engineering, Information
 Technology, Electronics & Control Engineering, Mechatronics, Computer
 Science & Systems Engineering, Electronics & Telematics, Metallurgy &
 Material Technology, Electronics & Computer Engineering, Production
 Engineering, Aeronautical Engineering, Instrumentation & Control
 Engineering, Bio-Technology and Automobile Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Solve $y(2x^2y + e^x) dx = (e^x + y^3) dy$.
 (b) Suppose that an object is heated to 300° F and allowed to cool in a room whose air temperature is 80° F, if after 10 minutes the temperature of the object is 250° F What will be its temperature after 20 minutes. [8+8]
2. (a) Solve $(4D^2 - 4D + 1)y = 100$
 (b) Solve $(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}$. [8+8]
3. (a) Find three positive numbers whose sum is 100 and whose product is maximum.
 (b) If $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$ prove that 'c' of the Cauchy's generalized mean value theorem is the geometric mean of 'a' and 'b' for any $a > 0, b > 0$. [8+8]
4. (a) Find the radius of curvature of $\sqrt{a} = \sqrt{r} \cos \frac{\theta}{2}$ at (r, θ) .
 (b) Find the envelope of the straight line $\frac{x}{a} + \frac{y}{b} = 1$ where $a^2 + b^2 = 4$. [8+8]
5. (a) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dx dy dz$.
 (b) Find the surface area of the solid generated by revolving the arc of the parabola $x^2 = 12y$, bounded by its latus rectum about y-axis. [8+8]
6. (a) Examine the convergence of $\frac{1}{3}x^2 + \frac{1.2}{3.5}x^3 + \frac{1.2.3}{3.5.7}x^4 + \dots, (x > 0)$
 (b) Examine the convergence of $\sum \frac{[(n+1)!]^2 x^{n-1}}{n}, (x > 0)$ [8+8]
7. Verify Stoke's theorem for $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ over the box bounded by the planes $x = 0, x = a, y = 0, y = b, z = c$. [16]
8. (a) Using Laplace transform, solve $(D^2 + 2D - 3)y = \sin x, y(0) = y'(0) = 0$.

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(b) Using Laplace transform evaluate $\int_0^{\infty} (e^{-t} - e^{-2t})/t dt.$ [8+8]

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1. (a) Solve $x \frac{dy}{dx} + y = x^3 y^6$
 (b) A bacterial culture, growing exponentially, increases from 200 to 500 grams in the period from 6 a.m. to 9 a.m. How many grams will be present at noon. [8+8]

2. (a) Solve $(D^2 - 4D + 13) y = e^{2x}$
 (b) Solve $(D^2 + 16) y = e^{-4x}$. [8+8]

3. (a) Find the region in which $f(x) = 1 - 4x - x^2$ is increasing and the region in which it is decreasing using Mean Value Theorem .
 (b) Find the minimum value of $x^2 + y^2 + z^2$ given $x + y + z = 3a$. [8+8]

4. (a) Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$.
 (b) Find the equation of the circle of curvature of the curve
 $x = a(\cos \theta + \theta \sin \theta)$, $y = a(\sin \theta - \theta \cos \theta)$ [8+8]

5. (a) Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} dz dy dx$.
 (b) Find the volume of the solid that results when the region enclosed by the curve ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $(0 < b < a)$ rotates about major axis. [8+8]

6. (a) Test the convergence of $\sum (\sqrt{n^3 + 1} - \sqrt{n^3})$
 (b) Test the convergence of $\sum (x^n / n^{n-1})$, $(x > 0)$ [8+8]

7. Verify Stoke's theorem for $\vec{F} = (2x - y) \vec{i} - yz^2 \vec{j} - y^2 z \vec{k}$ where S is the upper half surface $x^2 + y^2 + z^2 = 1$ of the sphere and C is its boundary. [16]

8. (a) Using Laplace transform, evaluate $\int_0^{\infty} \frac{(\cos at - \cos bt)}{t} dt$.

(b) Using Laplace transform, solve $y(t) = 1 - e^{-t} + \int_0^t y(t-u) \sin u du$. [8+8]

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1. (a) Solve $x^3 \sec^2 y \frac{dy}{dx} + 3x^2 \tan y = \cos x$.
 (b) A bacterial culture, growing exponentially, increases from 100 to 400 grams in 10 hours. How much was present after 3 hours? [8+8]
2. (a) Solve $(4D^2 - 4D + 1)y = 100$
 (b) Solve $(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}$. [8+8]
3. (a) Find the maxima and minima of the function $f(x) = 2(x^2 - y^2) - x^4 + y^4$.
 (b) Prove using Mean Value theorem $|\sin u - \sin v| \leq |u - v|$. [8+8]
4. (a) Find the circle of curvature at $(0, 0)$ for $x + y = x^2 + y^2 + x^3$.
 (b) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [8+8]
5. (a) Find the volume of the solid when ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $(0 < b < a)$ rotates about minor axis.
 (b) By transforming into polar coordinates evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dx dy$ over the annular region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$, with $b > a$. [8+8]
6. (a) Examine the convergence of $\sum 1/n(2n + 1)$
 (b) Examine the convergence of $\frac{1}{1.3.5} - \frac{1}{3.5.7} + \frac{1}{5.7.9} - \frac{1}{7.9.11} + \dots$ [8+8]
7. Evaluate $\iint_S \vec{F} \cdot d\vec{S}$, if $\vec{F} = yz \vec{i} + 2y^2 \vec{j} + xz^2 \vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = 9$ contained in the first octant between the planes $z = 0$ and $z = 2$. [16]
8. (a) Using Laplace transform, solve $(D^2 + 5D - 6)y = x^2 e^{-x}$, $y(0) = a$, $y'(0) = b$.

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(b) Using Laplace transform, evaluate $\int_0^{\infty} [(\cos 5t - \cos 3t) / t] dt.$ [8+8]

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1. (a) Solve $x dx + y dy = \frac{a^2(xdy-ydx)}{x^2+y^2}$.
 (b) The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after $1\frac{1}{2}$ hours? [8+8]
2. Solve $(D^2 + 1)y = \sin x \sin 2x + e^x x^2$. [16]
3. (a) Using Rolle's theorem show that $g(x) = 8x^3 - 6x^2 - 2x + 1$ has a zero between 0 and 1.
 (b) If $u = \frac{yz}{x}$, $v = \frac{xz}{y}$, $w = \frac{xy}{z}$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. [8+8]
4. (a) Find the radius of curvature of $x^3 + y^3 = 3axy$ at $(\frac{3a}{2}, \frac{3a}{2})$.
 (b) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$ where $a^m b^n = c^{m+n}$. [8+8]
5. (a) By changing the order of integration, evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2} dy dx$.
 (b) Evaluate $\iiint_R (x+y+z) dz dy dx$ where R is the region bounded by the planes $x=0, x=1, y=0, y=1, z=0, z=1$. [8+8]
6. (a) Examine the convergence or divergence of $\sum x^{2n}/(n+2)\sqrt{(n+2)}$, ($x > 0$)
 (b) Examine the convergence of $\sum 1/(n^{3/2} + n + 1)$ [8+8]
7. (a) Prove that $grad(\vec{F} \cdot \vec{G}) = \vec{F} \times (\nabla \times \vec{G}) + \vec{G} \times (\nabla \times \vec{F}) + (\vec{F} \cdot \nabla)\vec{G} + (\vec{G} \cdot \nabla)\vec{F}$
 (b) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = 3xy\vec{i} - y^2\vec{j}$ and C is the parabola $y = 2x^2$ from (0,0) to (1,2) [8+8]
8. (a) Find $L^{-1} [e^{-2s}/(s^2 + 4s + 5)]$

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Set No. 4

(b) Using Laplace transform, evaluate $\int_0^{\infty} e^{-at} \sin^2 t / t \, dt$. [8+8]
